Definition of Signal

☺A signal is a function of time representing a physical variable, e.g. voltage, current, spring displacement, share market prices, number of student asleep in the Lab, cash in the bank account.

☺Typically we will use a mathematical function such as f(t), u(t) or y(t) to describe a signal which is a continuous function of time.

Signal Classification

Signals may be classified into:

1. Continuous-time and discrete-time signals
2. Analogue and digital signals
3. Periodic and aperiodic signals
4. Energy and power signals
5. Deterministic and probabilistic signals
6. Causal and non-causal
7. Even and Odd signals

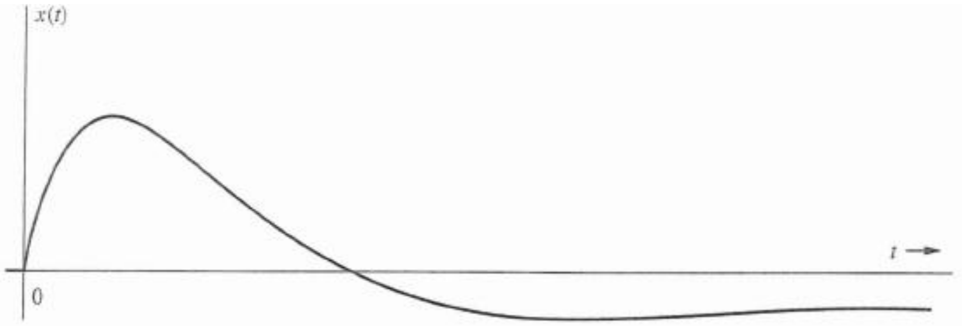
Classification of signals

☺A **continuous-time** (or analog) signal exists at all instants of time. The real word consists of continuous signals, and are usually written as a function of t.

☺A **discrete-time** signal exists only at discrete instants of time and is usually derived from a continuous signal by the process of sampling, e.g. measuring the temperature at 3 o'clock in the afternoon.

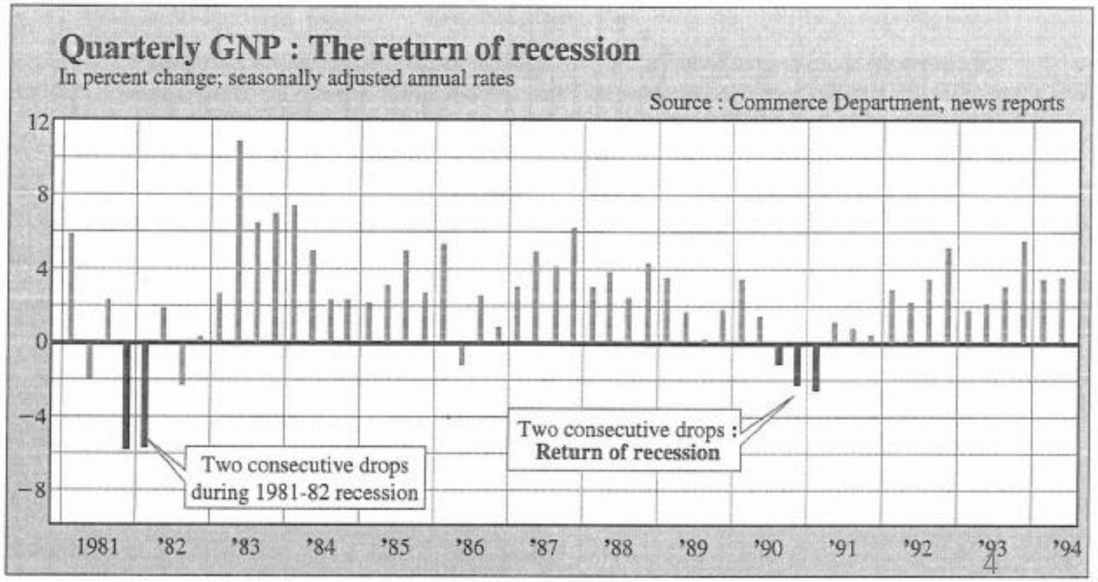
Signal Classification- Continuous vs Discrete

Continuous-time



(a)

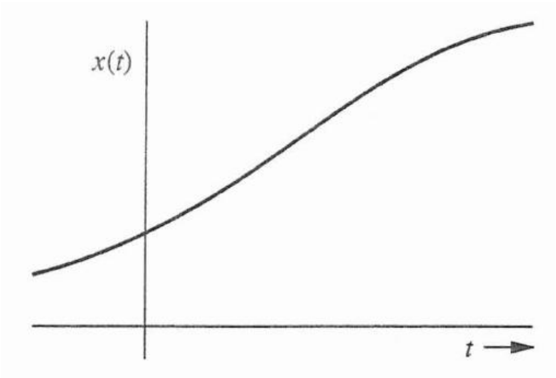
Discrete-time



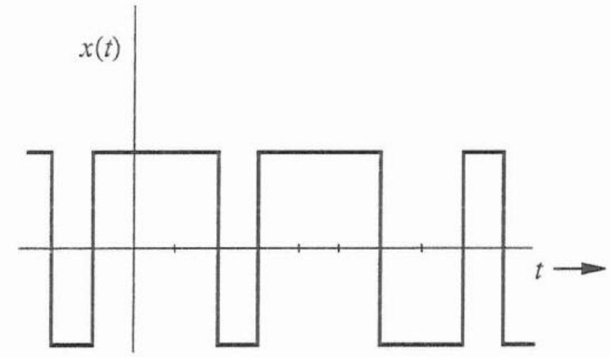
(b)

Signal Classification- Analogue vs Digital

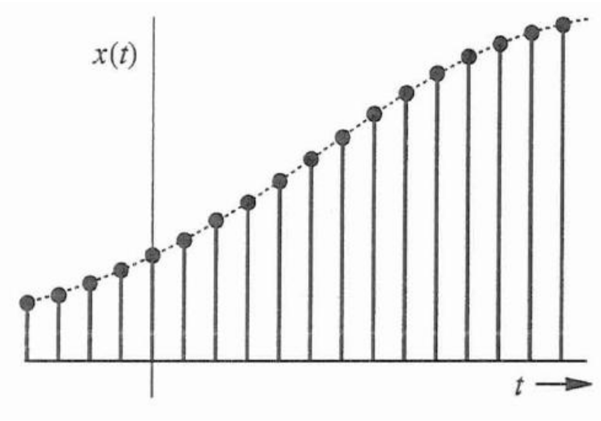
Analogue, continuous



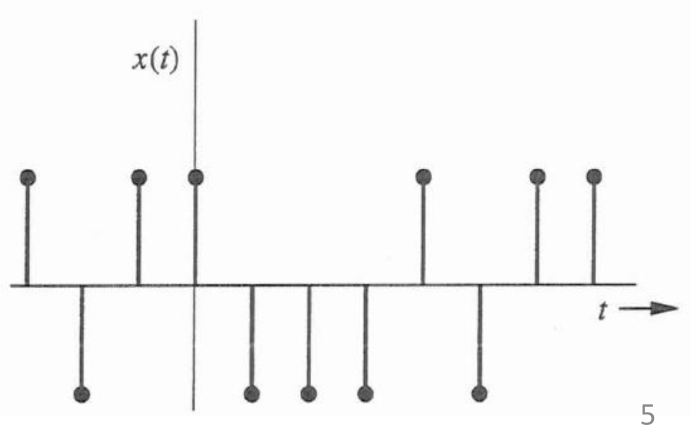
Digital, continuous



Analogue, discrete



Digital, discrete



Signal Classification

- Periodic and non-periodic signals
- A signal is periodic if

$$x(t) = x(t+T)$$

where T is the period.

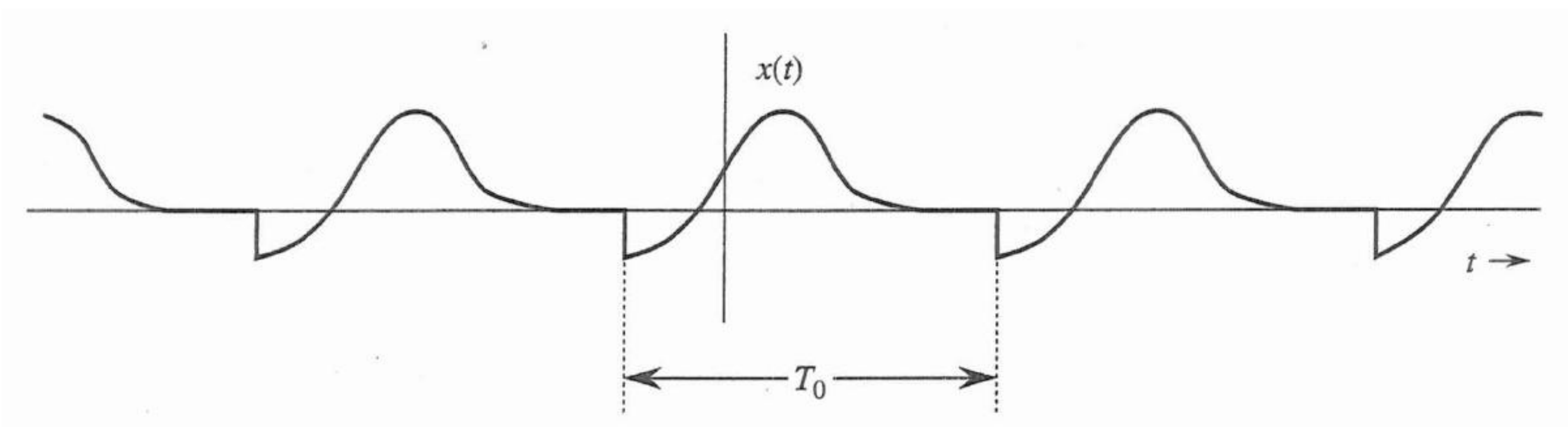
- **Non-periodic signals are those which doest not follow this euation**

$$x(t) \neq x(t+T)$$

A signal *x(t) is said to be periodic if for some positive constant* T_o

$$x(t) = x(t+T_o) \quad \text{for all } t$$

The smallest value of T_o *that satisfies the periodicity condition of this* equation is the *fundamental period of x(t).*



Signal Classification- Energy v/s Power

- Energy of a signal *x(t)* is given by:

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

- Power of a signal *x(t)* is given by:

$$P_x = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

- A signal is Energy signal if $0 < Ex < \infty$
- A signal is Power signal if $0 < Px < \infty$

Energy Signal and Power Signal

- Consider applying a time varying voltage $v(t)$ to a resistor,

  the instantaneous average power drops across the resistor is,

$$p(t) = v^2(t) / R$$

or

$$p(t) = i^2(t) \times R$$

Power Signal

- For the purposes of signal classification the average power of a signal is measured over all time, i.e., $T \to \infty$

so

$$average\ power = \lim_{T \to \infty} \left(\frac{1}{T} \int_{t=-T/2}^{T/2} y^2(t)\, dt \right)$$

Energy Signal

- To get energy in Joules (J), it is necessary to integrate over some specified time interval.
- So the energy in a signal between time 0 and time T:

$$energy(T) = \int_{t=0}^{T} y^2(t)\,dt$$

- and thus the total energy is

$$total\ energy = \int_{t=-\infty}^{\infty} y^2(t)\,dt$$

Relation of Power and Energy Signal

- the average power can be expressed as:

$$average\ \ power = \lim_{T \to \infty} \left(\frac{1}{T} \times total\ \ energy \right)$$

- Clearly, because of the $1/T$ factor, if the total energy is finite then the average power is zero.
- Conversely, if the average power is not zero then the total energy is infinite

Energy Signal

- if the resistor is 1 Ω , the power is equal to the square of the voltage or current signal. In general, the instantaneous power of a signal is taken to be the square of the signal:

$$p(t) = y^2(t)$$

- The average power of a signal is its mean value. For instance, over a time period -*T*/2 to *T*/2

$$average\ power = \frac{1}{T} \int_{t=-T/2}^{T/2} y^2(t)\,dt$$

Example – Finite Energy Signal

- Consider a transient signal that starts at $t = 0$ and decays to zero with an exponential form:

$$y(t) = e^{-t}, \quad t \geq 0$$

$$total\ energy = \int_{t=-\infty}^{\infty} y^2(t)\,dt = \int_{t=0}^{\infty} e^{-2t}\,dt = \frac{1}{2}$$

which is a <u>finite energy signal</u>

Example - Non-zero Average Power Signal

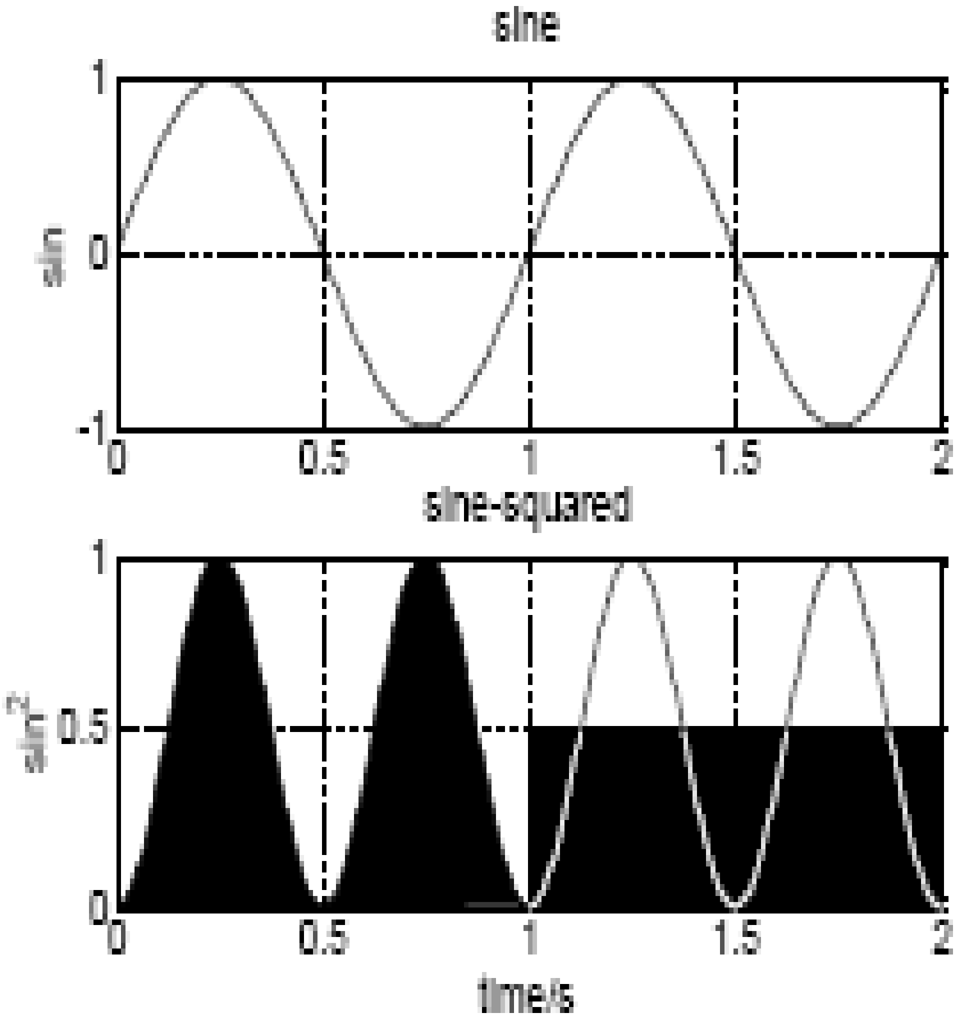
- The average power of a periodic signals can be written as, (WHY?)

$$\overline{P} = \frac{1}{T} \int_{t=0}^{T} y^2(t)\, dt$$

- Let $y(t) = \sin(\omega t)$, where T is the period equals $2\pi/\omega$

$$\overline{P} = \frac{1}{T} \int_{t=0}^{T} \sin^2\left(\frac{2\pi}{T} t\right) dt = \frac{1}{2}$$

Example



Deterministic Signals

– Everything is known about the signal

– $x(t) = A\sin(wt)$, $\{A,w\}$ known parameters, no noise or unknown parameters

Random Signals

– Some property or parameter of the signal is unknown

– May be completely random such as additive noise $n(t)$

– random process

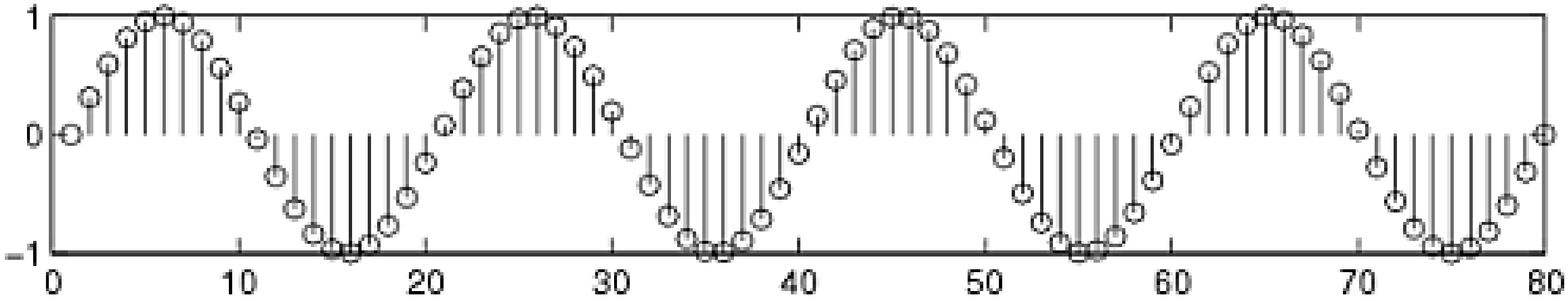
– Deterministic signal with additive noise eg. $r(t) = x(t) + n(t)$

– Random parameter of a deterministic signal eg. $x(t) = A\sin(wt)$, A is random number

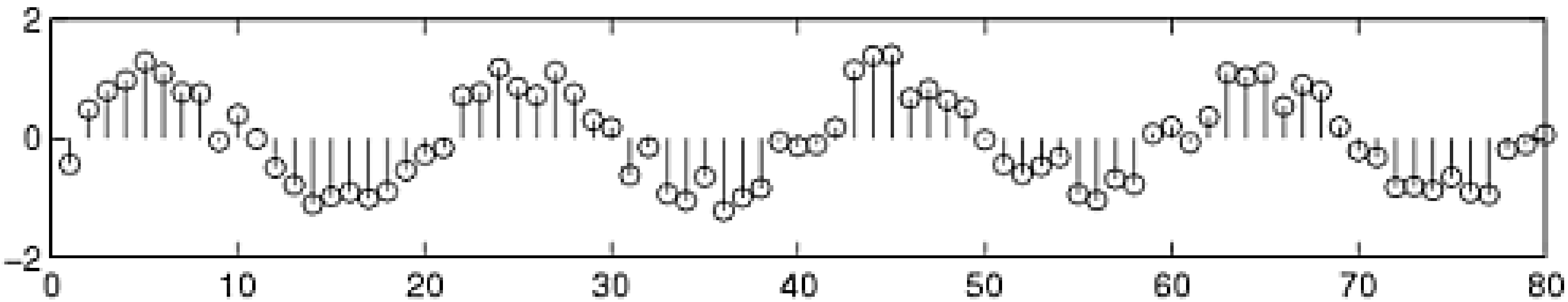
– $r(t)$, $n(t)$, $x(t)$ all random processes

Signal Classification- Deterministic vs Random

Deterministic

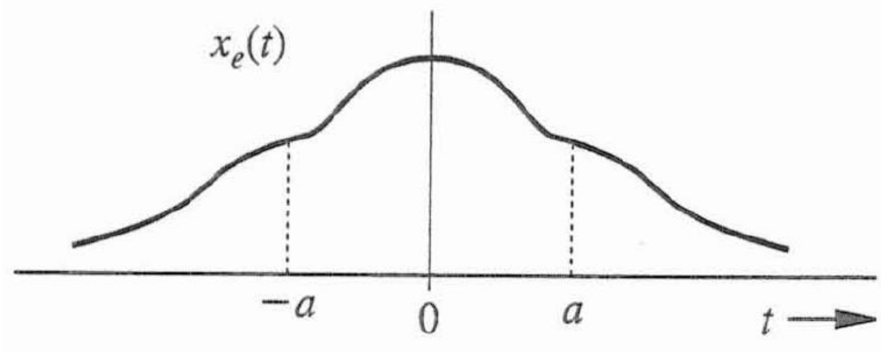


Random



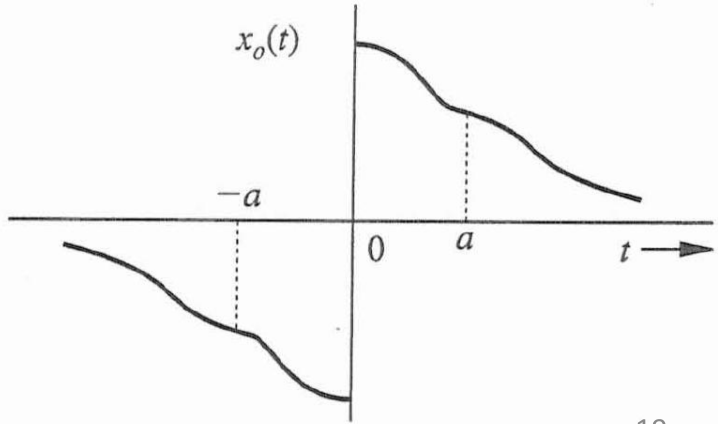
A real function $x_e(t)$ *is said to be an even function of t if*

$$x_e(t) = x_e(-t)$$

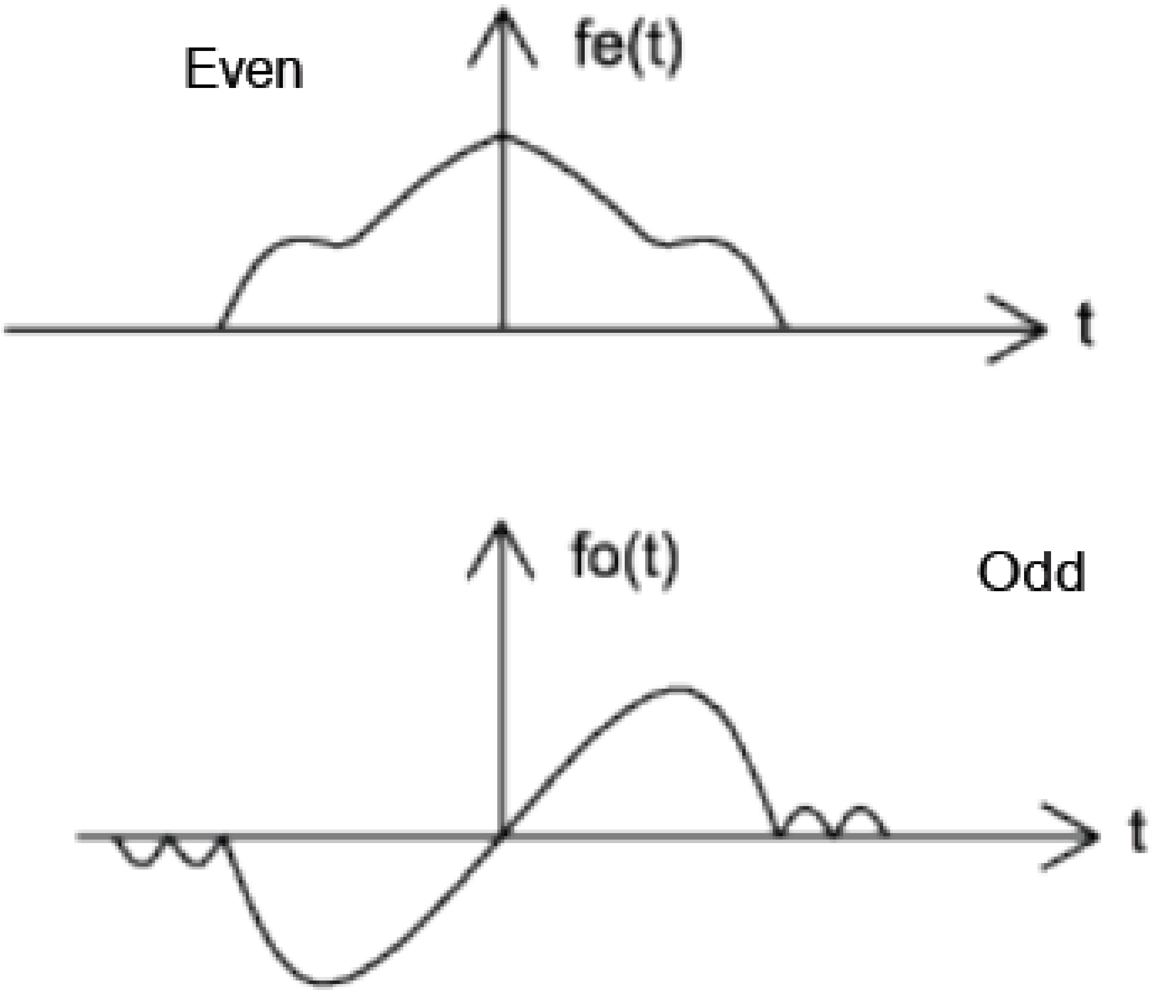


A real function $x_o(t)$ *is said to be an odd function of t if*

$$x_o(t) = -x_o(-t)$$



Signal Classification- Even vs Odd



Every signal *x(t) can be expressed as a sum of even and* odd components because:

$$x(t) = \underbrace{\tfrac{1}{2}[x(t) + x(-t)]}_{\text{even}} + \underbrace{\tfrac{1}{2}[x(t) - x(-t)]}_{\text{odd}}$$